Disturbance observer-based sliding mode control for multi-agent systems with mismatched uncertainties

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Abstract

Purpose – This paper aims to develop sliding mode control (SMC) methods for second-order multi-agent systems (MAS) in the presence of mismatched uncertainties.

Design/methodology/approach – Based on the disturbance observer (DOB), discontinuous and continuous sliding mode protocols are designed to achieve finite-time consensus in spite of the disturbances.

Findings – Compared with integral SMC, numerical simulation results show that the proposed control methods exhibit better performance with respect to reduction of chattering.

Originality/value – The main contributions are the following: MAS described with mismatched uncertainties are considered; both discontinuous and continuous sliding mode controllers are considered; with the proposed sliding mode controller, the desired sliding surface can be reached in finite time and the DOB is introduced in the controller to alleviate the chattering phenomenon.

Keywords Sliding mode control, Multi-agent system, Chattering reduction, Disturbance observer, Finite-time consensus, Mismatched uncertainties

Paper type Research paper

1. Introduction

In recent years, the interest in cooperative control of multi-agent systems (MAS) has been growing greatly among researchers. Its broad application has rapidly developed fields such as physics, sociology, biology, artificial intelligence, sensor networks and control engineering. Consensus is the fundamental problem of cooperative control, which aims to design control laws to make certain variables of concern reach an agreement, as reported by Olfati-Saber and Murray (2004), Ren and Atkins (2007), Lin and Jia (2009), Li and Zhang (2010), Cheng *et al.* (2016), Liu *et al.* (2017), Wang *et al.* (2018). With further study on cooperative problems, the scope of research has greatly widened to include tracking (Hong *et al.*, 2006; Li *et al.*, 2013; Xu *et al.*, 2014; Cheng *et al.*, 2010), formation (Liu and Tian, 2009; Dong and Hu, 2016), containment control (Wang *et al.*, 2014; Sun *et al.*, 2017), flocking (Olfati-Saber, 2006; Yu *et al.*, 2010), etc.

Because of the advantages of robustness and simplicity, sliding mode control (SMC) is widely used for nonlinear systems. A lot of derivative methods have sprung up including feedback linearization control (Li *et al.*, 2016), terminal SMC (Yan *et al.*, 2016; Mu *et al.*, 2016), high-order SMC (Yan *et al.*, 2016), adaptive SMC and so on. In Lu *et al.* (2012), the sliding mode controller, combined with adaptive algorithm, was designed for attitude tracking control issues of a nonlinear spacecraft model with external disturbances and uncertainties in inertia. In Li *et al.* (2016), a new fixed-time SMC algorithm using the backstepping method was proposed for a class of high-order strict-feedback nonlinear systems (SFNSs) with mismatching system uncertainties. In Yan *et al.* (2016), Euler's discretization of the second-order SMC system with the twisting algorithm was studied. In Yang *et al.* (2014), a continuous dynamic sliding mode control method was proposed for mismatched disturbance attenuation using a high-order sliding mode differentiator.

Because of its particular robustness to restrain disturbances and plant uncertainties, the SMC is widely used in MAS. In Ren and Chen (2015), both a new distributed asymptotic consensus controller and terminal SMC were considered for the leader-following consensus problem of second-order nonlinear MAS. In Yu and Long (2015), both the discontinuous or continuous integral sliding mode protocols were developed to achieve accurate finite-time consensus in spite of the disturbances for the

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second-order MAS. In Han *et al.* (2017), the distributed finite-time formation tracking protocols were proposed via the fast terminal SMC scheme for finite-time formation tracking control problems of MAS.

Chattering phenomenon is inevitable in SMC. To improve control performance, some advanced control methods, such as neural networks, fuzzy control, were combined with SMC. In Zou *et al.* (2013), Chebyshev neural networks were used in conjunction with terminal SMCs. In Chang *et al.* (2012), a fuzzy sliding-mode formation controller was proposed to address the decentralized formation problems for multiple robots. However, chattering attenuation was achieved at the price of sacrificing the control performance.

Compared with the methods above, a disturbance observer (DOB) serves like a patch to the baseline controller and does not cause any adverse effects in the absence of uncertainties (Yang *et al.*, 2013; Zhang *et al.*, 2016). Besides, it can not only handle mismatched uncertainties but also has the advantage of simplicity. Therefore, the DOB is introduced for SMC in the paper. In conjunction with the DOB, a finite time sliding mode controller is proposed for second-order MAS with mismatched uncertainties in this paper.

In our paper, with the proposed sliding mode controller, the main characteristics are listed:

- MAS described with mismatched uncertainties are considered;
- both discontinuous and continuous sliding mode controllers are considered; furthermore, comparisons among integral, discontinuous and continuous sliding mode controllers are made to show the advantages of the proposed method;
- with the proposed sliding mode controller, the desired sliding surface can be reached in finite time;
- the SMC via a DOB in this paper could attenuate the mismatched disturbances without sacrificing its nominal performance and the chattering problem can be relieved to some extent.

The paper is organized as follows: graph theory and preliminaries of SMC are introduced in Section 2. In Section 3, on the basis of the DOB, both discontinuous and continuous sliding mode controllers are designed to estimate the uncertainties and achieve consensus. Then, numerical simulation examples are shown to illustrate the analytical results in Section 4. Eventually, Section 5 gives a brief conclusion to this paper.

Notation: In the following sections, the vector $x = [x_1,\ldots, x_N]^T$, $\alpha \in R$ and $sgn(\cdot)$ denotes the sign function. Define the function $sig^{\alpha}(x_i) = |x_i|^{\alpha} sgn(x_i)$, $i = 1,\cdots,N$, and the vector $sig^{\alpha}(x) = [sig^{\alpha}(x_1),\ldots,sig^{\alpha}(x_N)]^T$; $\mathbf{0}$ and $\mathbf{1}_N$ represent the column vectors with all elements being 0 and 1, respectively. For vectors, $||\cdot||_1$ and $||\cdot||_2$ denote the 1-norm and Euclidean norm, respectively. For instance, $||x||_1 = \sum_{i=1}^{N} |x_i|$, $||x||_2 = \sqrt{x^T x}$.

2. Preliminaries and problem formulation

2.1 Graph theory

For a graph $\mathcal{G}$ with n vertices, denoting the graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$ represents the nodes, and $\mathcal{E} = \{(v_i, v_j) | v_i, v_j \in \mathcal{V}\}$ is the set of edges. $\mathcal{G}$ is called undirected if $(v_i, v_j) \in \mathcal{E} \Longleftrightarrow (v_j, v_i) \in \mathcal{E}$. Meanwhile, if there is an edge between v_i and v_j, then it is said that node v_i and node v_j are adjacent. The adjacency matrix $\mathcal{A} = [a_{ij}] \in R^{n\times n}$ associated with $\mathcal{G}$ is defined as $a_{ij} = 1$ if $(v_j, v_i) \in \mathcal{E}$. For each node v_i, N_i is the cardinality of its neighbor set and $N_m = \max_{i=1,\cdots,n} \{N_i\}$.

2.2 Problem formulation

Consider the second-order MAS with mismatched disturbance, depicted by:

$$\begin{cases} \dot{x}_i(t) = v_i(t) + d_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \tag{1}$$

where x_i and $v_i(t)$ represent the position and velocity, respectively, $u_i(t)$ is the control input, $d_i(t)$ is the disturbance.

Assumption 1. For MAS (1), there exists a directed spanning tree in graph $\mathcal{G}$.

Assumption 2. The disturbance in system (1) is bounded by $d^* = \sup_{t>0} |d_i(t)|$ and satisfies $\lim_{t\to\infty} \dot{d}_i(t) = 0$.

Lemma 1. For $x_i \in R$, $i = 1,2,\ldots,n$, $\alpha \in (0,1]$, then $\left(\sum_{i=1}^{n} |x_i|\right)^{\alpha} \le \sum_{i=1}^{n} |x_i|^{\alpha}$.

Lemma 2. (Finite-time Lyapunov Stability Theorem) Consider the non-Lipschiz continuous nonlinear system $\dot{x} = f(x)$ with $f(0) = 0$. Suppose there exists a continuous function $V(x)$ defined on a neighborhood of the origin, and real numbers $c > 0$ and $0 < \alpha < 1$, such that the following conditions hold:

- $V(x)$ is positive definite;
- $\dot{V}(x) + cV^{\alpha} \le 0$.

Then the origin is locally finite-time stable, and the settling time, depending on the initial state $x(0) = x_0$, satisfies:

$$T(X_0) \le \frac{1}{c(1-\alpha)} V(x_0)^{1-\alpha}$$

for all x_0 in some open neighborhood of the origin.

3. Main results

3.1 Integral sliding mode control

Integral SMC is an effective method for restraining the mismatched uncertainties. Motivated by the integral sliding mode method (Sam *et al.*, 2004), the following sliding mode surface is selected:

$$s_i = v_i + c_1 \sum_{j=1}^{N} a_{ij}(x_i - x_j) + c_2 \sum_{j=1}^{N} \int a_{ij}(x_i - x_j) \tag{2}$$

The integral SMC controller is designed as:

$$u_i(t) = -\left[c_1 \sum_{j=1}^{N} a_{ij}(v_i - v_j) + c_2 \sum_{j=1}^{N} a_{ij}(x_i - x_j) + ksgn(s_i) \right] \tag{3}$$

We can derive the derivative of s_i,

$$\begin{aligned}\dot{s}_i &= u_i + c_1\sum_{j=1}^{N} a_{ij}(v_i - v_j) + c_1\sum_{j=1}^{N} a_{ij}(d_i - d_j) + c_2\sum_{j=1}^{N} a_{ij}(x_i - x_j)\\ &= -ksgn(s_i) + c_1\sum_{j=1}^{N} a_{ij}(d_i - d_j)\end{aligned} \tag{4}$$

From (4), we can design the switching gain $k > c_1 d^\star$ so that the sliding surface s_i can reach zero in finite time. Then, substituting the condition $s_i = 0$ into (2), yields:

$$v_i = -c_1\sum_{j=1}^{N} a_{ij}(x_i - x_j) - c_2\sum_{j=1}^{N}\int a_{ij}(x_i - x_j) \tag{5}$$

Then combining (1) and (5), we can derive that:

$$x_i + c_1\sum_{j=1}^{N} a_{ij}(\dot{x}_i - \dot{x}_j) + c_2\sum_{j=1}^{N} a_{ij}(x_i - x_j) = \dot{d}_i \tag{6}$$

Therefore, if $\lim_{t\to\infty}\dot{d}(t) = 0$, it is easy to derive that $x_i \to x_j$ and $v_i \to v_j$.

Lemma 3. (Ren and Atkins, 2007) For the second-order MAS:

$$\begin{cases}\dot{x}_i(t) = v_i(t)\\ \dot{v}_i(t) = u_i(t)\end{cases} \tag{7}$$

With the control protocol $u_i(t) = -c_1\sum_{j=1}^{N} a_{ij}(v_i - v_j) - c_2\sum_{j=1}^{N} a_{ij}(x_i - x_j)$, the system (7) can achieve consensus, and the consistent function satisfies:

$$\lim_{t\to\infty}\left(x(t) - \mathbf{1}p^T x(0) - t\mathbf{1}p^T v(0)\right) = 0,$$

$$\lim_{t\to\infty}\left(v(t) - \mathbf{1}p^T v(0)\right) = 0,$$

where $\mathbf{1} = [1,1,\ldots,1]^T$ and p is the non-negative left eigenvector of $-L$ corresponding to eigenvalue 0 and $p^T\mathbf{1} = 1$.

Remark 1. If $\lim_{t\to\infty}\dot{d}(t) = 0$, equation (6) is equivalent to the above system (7), then we can have that $\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0$ and $\lim_{t\to\infty}\left(v_i(t) - v_j(t)\right) = 0$.

Though the integral sliding mode method is efficient to restrain the mismatched uncertainties, it always introduces overshooting to the control performance; besides, the integral SMC does not do well in alleviating the chattering phenomenon. To illustrate these, the simulation results will be shown in Section 4.

3.2 Discontinuous sliding mode control via a disturbance observer

Instead of the integral SMC, a new discontinuous sliding mode controller via a DOB is proposed to alleviate the chattering phenomenon and improve the control performance in this section.

Define $z_i = [x_i, v_i]^T$, then the system (1) can be described as:

$$\dot{z}_i = f_i(v_i) + G_1 u_i + G_2 d_i \tag{8}$$

where $f_i(v_i) = [v_i, 0]^T$, $G_1 = [0,1]^T$, $G_2 = [1,0]^T$.

Then the nonlinear DOB is introduced, which is described as the following:

$$\begin{cases}\dot{p}_i = -LG_2 p_i - L\left[G_2 L z_i + f_i(v_i) + G_1 u_i\right]\\ \hat{d}_i = p_i + L z_i\end{cases} \tag{9}$$

where $\hat{d}_i$, p_i, and L represent the estimation of nonlinear disturbance, the internal state of the nonlinear observer and the observer gain, respectively.

A novel sliding mode surface for system (1) with nonlinear uncertainties is selected as the following:

$$s_i = v_i + c\sum_{j=1}^{N} a_{ij}(x_i - x_j) + \hat{d}_i \tag{10}$$

where $\hat{d}_i$ is the estimation of disturbance d_i.

The proposed sliding mode controller based on DOB is designed as:

$$u_i = -\left[c\sum_{j=1}^{N} a_{ij}(v_i - v_j) + c\sum_{j=1}^{N} a_{ij}\left(\hat{d}_i - \hat{d}_j\right) + ksgn(s_i)\right] \tag{11}$$

Therefore, substituting u_i into the derivation of s_i,

$$\begin{aligned}\dot{s}_i &= \dot{v}_i + c\left[\sum_{j=1}^{N} a_{ij}(v_i - v_j) + \sum_{j=1}^{N} a_{ij}\left(\hat{d}_i - \hat{d}_j\right)\right] + \dot{\hat{d}}_i\\ &= -c\sum_{j=1}^{N} a_{ij}\left(\hat{d}_i - \hat{d}_j\right) + c\sum_{j=1}^{N} a_{ij}(d_i - d_j) + \dot{\hat{d}}_i - ksgn(s_i)\end{aligned} \tag{12}$$

According to (8) and (9), it derives that:

$$\begin{aligned}\dot{\hat{d}}_i &= \dot{p}_i + L\dot{z}_i\\ &= -LG_2 p_i\\ &\quad - L\left[G_2 L z_i + f_i(v_i) + G_1 u_i\right] + L\left[f_i(v_i) + G_1 u_i + G_2 d_i\right]\\ &= -LG_2(p_i + L z_i) + LG_2 d_i = -LG_2\hat{d}_i + LG_2 d_i\end{aligned} \tag{13}$$

Substituting (13) into (12), it follows that:

$$\dot{s}_i = -ksgn(s_i) + Lg_2 e_{d_i} + c\sum_{j=1}^{N} a_{ij}(e_{d_i} - e_{d_j}) \tag{14}$$

where $e_{d_i} = d_i - \hat{d}_i$ and $e_{d_j} = d_j - \hat{d}_j$.

Remark 2. The disturbance estimation error $e_{d_i} = d_i - \hat{d}_i$ is bounded, satisfying $e_d^* = \sup_{t>0}|e_{d_i}|$, $i = 1, \cdots, N$.

Proof: From the definition of e_{d_i} and equation (13), one can derive

$$\begin{aligned}\dot{e}_{d_i} &= \dot{d}_i - \dot{\hat{d}}_i\\ &= \dot{d}_i + LG_2\hat{d}_i - LG_2 d_i\\ &= -LG_2 e_{d_i} + \dot{d}_i\end{aligned} \tag{15}$$

It can be verified that the error system (15) is asymptotically stable because $\lim_{t\to\infty} \dot{d}_i = 0$ is satisfied in Assumption 2. Therefore, Remark 2 is rational.

Theorem 1. Suppose that Assumptions 1-3 hold for the MAS (1) and sliding mode surface (10), the proposed DOB (9) and discontinuous controller (11) guarantee that the sliding mode surface (10) is reached in finite divergence time, and then the MAS (1) sequentially slides along it to reach consensus asymptotically.

Proof: Consider the Lyapunov function as the following:

$$V_1 = \frac{1}{2} s^T s \tag{16}$$

The derivative of V_1,

$$\begin{aligned}\dot{V}_1 &= \sum_{i=1}^{N} \dot{s}_i s_i = \sum_{i=1}^{N} s_i \left[-ksgn(s_i) + LG_2 e_{d_i} + c\sum_{j=1}^{N} a_{ij}(e_{d_i} - e_{d_j}) \right] \\ &= \sum_{i=1}^{N} \left[-k|s_i| + LG_2 e_{d_i} s_i + cs_i \sum_{j=1}^{N} a_{ij}(e_{d_i} - e_{d_j}) \right] \\ &\leqslant -\sum_{i=1}^{N} \left[k - (2Nc + LG_2) e_d^* \right] |s_i| \\ &\leqslant -\sqrt{2}\left[k - (2Nc + LG_2) e_d^* \right] V_1^{\frac{1}{2}} \end{aligned} \tag{17}$$

According to the given condition $k > (2Nc + LG_2)e_d^*$, it is easy to derive that each agent can reach the sliding mode surface $s_i = 0$ in finite time. Therefore, from (10) we can derive:

$$\dot{x}_i = -c\sum_{j=1}^{N} a_{ij}(x_i - x_j) + d_i - \hat{d}_i \tag{18}$$

Combining (18) with the observer dynamics, yields:

$$\begin{cases} \dot{x}_i = -c\sum_{j=1}^{N} a_{ij}(x_i - x_j) + e_{d_i} \\ \dot{e}_{d_i} = -LG_2 e_{d_i} + \dot{d}_i \\ v_i = -c\sum_{j=1}^{N} a_{ij}(x_i - x_j) - \hat{d}_i \end{cases} \tag{19}$$

With the given condition that $c > 0$ and $LG_2 > 0$, it can be verified that the following system:

$$\begin{cases} \dot{x}_i = -c\sum_{j=1}^{N} a_{ij}(x_i - x_j) + e_{d_i} \\ \dot{e}_{d_i} = -LG_2 e_{d_i} \end{cases} \tag{20}$$

is exponentially stable. Because $\lim_{t\to\infty} \dot{d}_i = 0$, it can be derived that the system:

$$\begin{cases} \dot{x}_i = -c\sum_{j=1}^{N} a_{ij}(x_i - x_j) + e_{d_i} \\ \dot{e}_{d_i} = -LG_2 e_{d_i} + \dot{d}_i \end{cases} \tag{21}$$

is input-to-state stable. From the system (21), we can derive that $\lim_{t\to\infty} e_{d_i}(t) = 0$ and $\lim_{t\to\infty} \left(x_i(t) - x_j(t) \right) = 0$, implying that the system position can reach consensus asymptotically with the designed SMC method.

Remark 3. From system (20), we can have $\lim_{t\to\infty} e_{d_i}(t) = 0$; further, $\dot{x}_i = -c\sum_{j=1}^{N} a_{ij}(x_i - x_j)$ can be obtained; according to (Ren and Atkins, 2007), the position $x_i(t)$ can reach consensus satisfying $\lim_{t\to\infty} \left(x_i(t) - \sum_{i=1}^{N} \alpha_i x_i(0) \right) = 0$, where $\alpha = [\alpha_1, \ldots, \alpha_n]^T$ is a non-negative left eigenvector of $-L$ associated with eigenvalue 0 with $\alpha_i \geq 0$ and $\sum_{i=1}^{N} \alpha_i = 1$. Furthermore, the velocity $v_i(t)$ can reach consensus satisfying $\lim_{t\to\infty} v_i(t) = \lim_{t\to\infty} \dot{x}(t) = 0$.

3.3 Continuous sliding mode control via a disturbance observer

Using the fractional-order technique, we improve the discontinuous controller above and design a new continuous SMC in this section.

The continuous protocol for MAS (1) is as follows:

$$u_i = -\left[c\sum_{j=1}^{N} a_{ij}(v_i - v_j) + c\sum_{j=1}^{N} a_{ij}\left(\hat{d}_i - \hat{d}_j \right) + ksig^{1/2}(s_i) \right] \tag{22}$$

where $sig^{1/2}(s_i) = |s_i|^{1/2} sgn(s_i)$.

Theorem 2. With the fractional-order technique, the new proposed continuous controller (22) and DOB (9) can guarantee that the sliding mode surface (10) is reached in finite divergence time, and then the MAS (1) sequentially slides along it to reach consensus asymptotically.

Proof: Consider the Lyapunov function as the following:

$$V_2 = \frac{1}{2} \left[sig^{1/2}(s) \right]^T sig^{1/2}(s) \tag{23}$$

The derivative of V_2,

$$\begin{aligned}\dot{V}_2 &= \sum_{i=1}^{N} \frac{1}{2} |s_i|^{-\frac{1}{2}} \dot{s}_i sig^{1/2}(s_i) = \sum_{i=1}^{N} \frac{1}{2} sgn(s_i) \dot{s}_i \\ &= \sum_{i=1}^{N} \frac{1}{2} sgn(s_i) \left[-ksig^{1/2}(s_i) + LG_2 e_{d_i} + c\sum_{j=1}^{N} a_{ij}(e_{d_i} - e_{d_j}) \right] \\ &= -\frac{1}{2}\sum_{i=1}^{N} ksgn(s_i) sig^{1/2}(s_i) + \frac{1}{2}\sum_{i=1}^{N} sgn(s_i) \\ &\quad \left[c\sum_{j=1}^{N} a_{ij}(e_{d_i} - e_{d_j}) + LG_2 e_{d_i} \right] \\ &\leq -\frac{1}{2} k \sum_{i=1}^{N} |s_i|^{-\frac{1}{2}} sig^{1/2}(s_i) sig^{1/2}(s_i) \\ &\quad + k\left(CN^2 + \frac{1}{2} LG_2 \right) e_d^* \end{aligned} \tag{24}$$

In light of $|s_i|^{\frac{1}{2}} = |sig^{\frac{1}{2}}(s_i)| = V_{2i}^{\frac{1}{2}}$, the inequality (24) can be further deduced to

$$\begin{aligned}\dot{V}_2 &\le -\frac{1}{2}k\sum_{i=1}^{N} V_{2i}^{-\frac{1}{2}} V_{2i} + k\left(CN^2 + \frac{1}{2}LG_2\right)e_d^* \\ &= -\frac{1}{2}k\sum_{i=1}^{N} V_{2i}^{-\frac{1}{2}} + k\left(CN^2 + \frac{1}{2}LG_2\right)e_d^* \\ &\le -\frac{1}{2}kV_2^{\frac{1}{2}} + k\left(CN^2 + \frac{1}{2}LG_2\right)e_d^* \end{aligned} \tag{25}$$

Therefore, similar to Theorem 1, we can obtain that the system position and velocity will achieve consensus asymptotically with the continuous SMC method.

Remark 4. Throughout the three controllers, the integral SMC is effective to remove the offset asymptotically, but it always brings some adverse effects such as chattering. The DOB-based method helps to restrain the chattering phenomenon. The discontinuous SMC via a DOB suppresses the chattering, but the execution of the controller is discontinuous, which is not good in practice. To balance them, the improved continuous SMC via a DOB is designed. In fact, the continuous SMC via a DOB works better for the MAS (1).

4. Numerical example

Numerical simulations are given to verify the effectiveness of the theoretical result in this section. There are five agents, which are denoted by i = 1,2,. . .,5. The communication topology of the agents are given as in Figure 1.

In this example, we suppose there exist five agents for the MAS. The dynamics of each agent is described by MAS (1), and the nonlinear uncertainties $d_i(t)$ are described by $d_i(t) = e^{-t}$.

4.1 Integral sliding mode control

Consider MAS (1) with the integral sliding surface (2) and controller (3): Figures 2 and 3 describe the positions and velocities of the five agents, respectively. Figures 4 and 5 present the control inputs and sliding surfaces, respectively.

From Figures 2 and 3, it is easy to find that the integral sliding mode controller exhibits robustness in the presence of mismatched uncertainties. However, from Figure 4 it always brings adverse effects, such as overshoot and chattering.

Figure 1 The communication topology

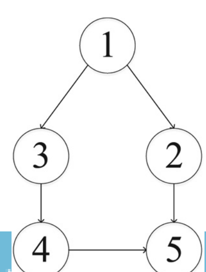


Figure 2 The position x_i of each agent

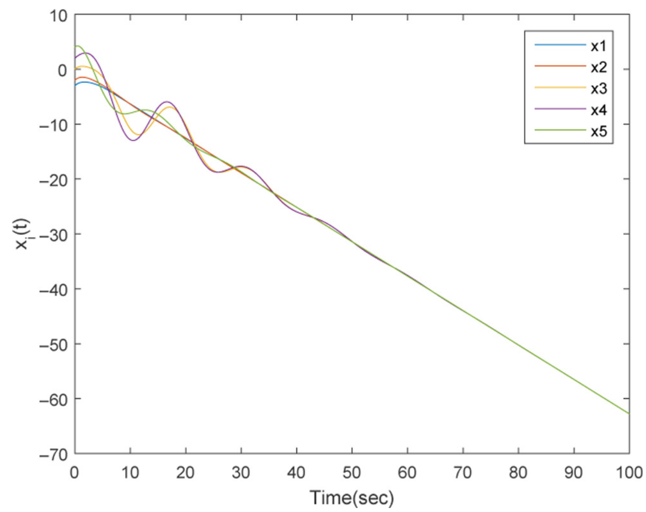
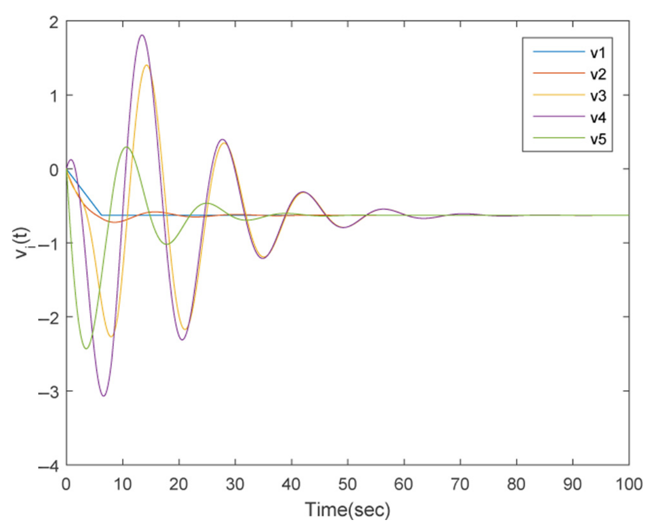


Figure 3 The velocity v_i of each agent



4.2 Discontinuous sliding mode control via disturbance observer

To improve the control performance, a DOB is introduced to alleviate the chattering phenomenon. With the proposed discontinuous sliding mode controller (11), Figures 6-9 are derived.

Obviously, the chattering phenomenon is alleviated with the DOB in comparison with the integral SMC.

4.3 Continuous sliding mode control via disturbance observer

To further improve the control performance, a new continuous SMC is designed with the fractional-order technique as (22) and Figures 10-13 are derived.

Figure 4 The input u_i of each agent

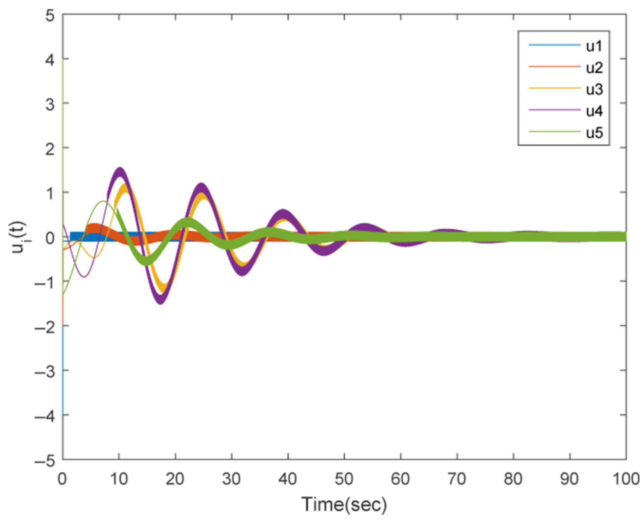
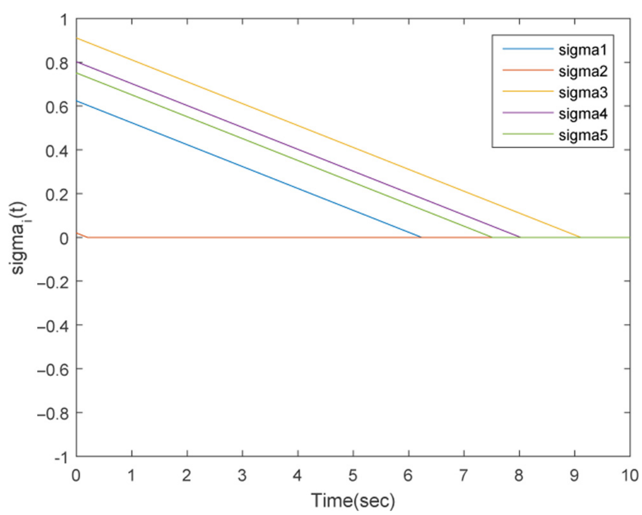


Figure 5 The sliding surface s_i of each agent



4.4 $d_i(t) = e^{-t} + v_i(t)\sin(v_i(t))$ with continuous SMC via disturbance observer

In practice, the uncertainties are mostly related to the velocities of the system. Therefore, in this section we consider the uncertainties $d_i(t) = e^{-t} + v_i(t)\sin(v_i(t))$ and derive Figures 14-17 with the continuous SMC via a DOB.

From the simulation results, it is obvious to see that the continuous SMC still works well to achieve consensus for MAS (1).

5. Conclusion

This paper establishes finite-time SMC protocols for group consensus to deal with MAS with mismatched uncertainties. In conjunction with a DOB, the uncertainties are estimated and chattering phenomenon is alleviated. In addition, both discontinuous and continuous sliding mode controllers are proposed in the paper. A numerical simulation is shown to

Figure 6 The position x_i of each agent

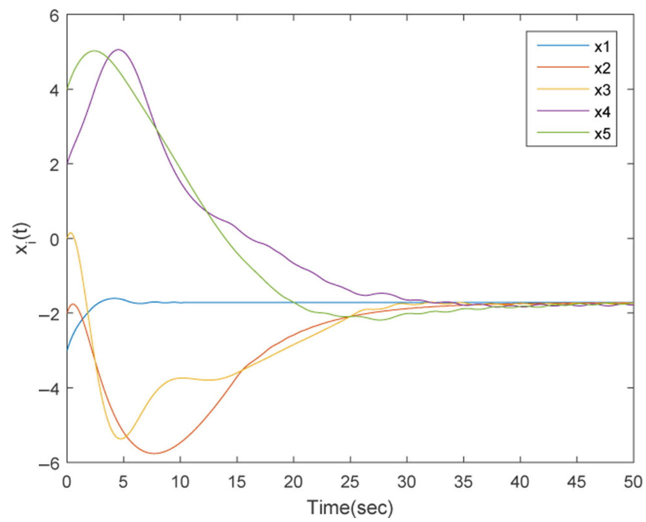


Figure 7 The velocity v_i of each agent

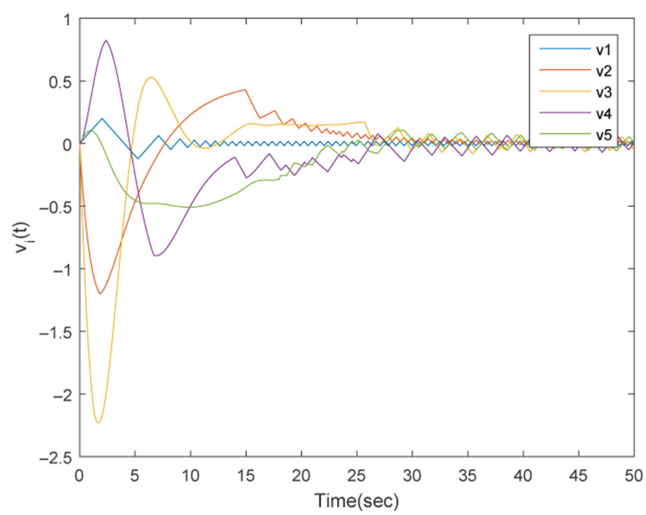


Figure 8 The input u_i of each agent

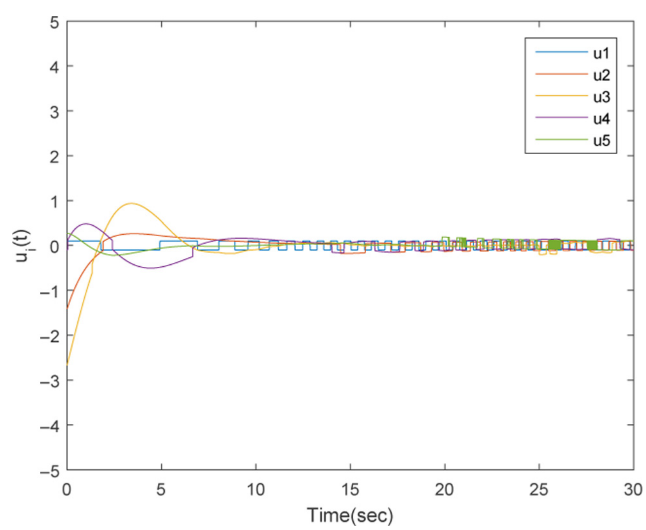


Figure 9 The sliding surface s_i of each agent

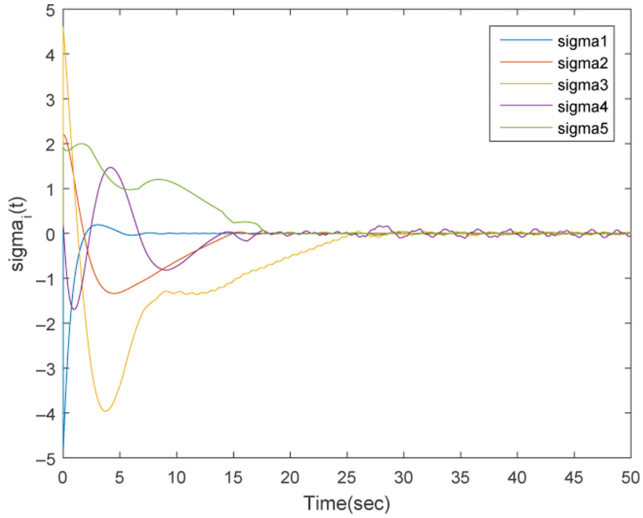


Figure 10 The position x_i of each agent

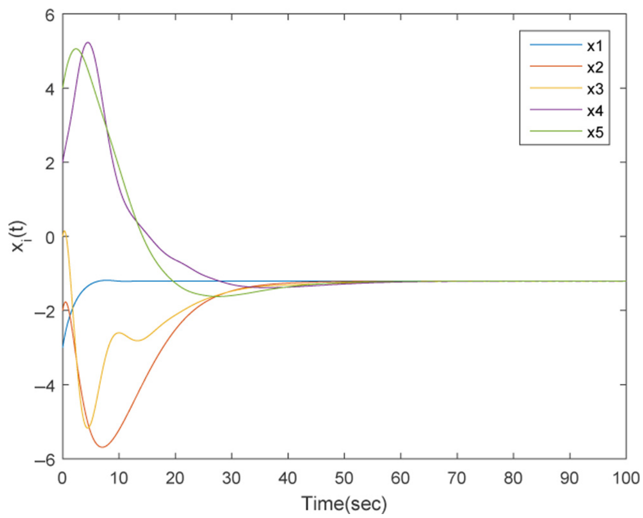


Figure 11 The velocity v_i of each agent

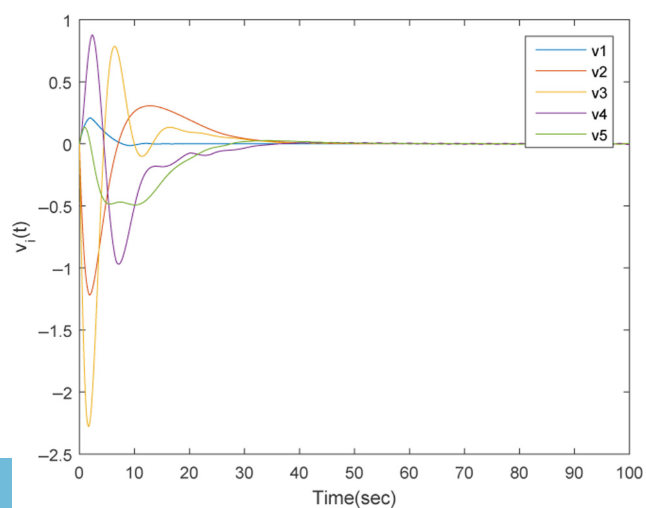


Figure 12 The input u_i of each agent

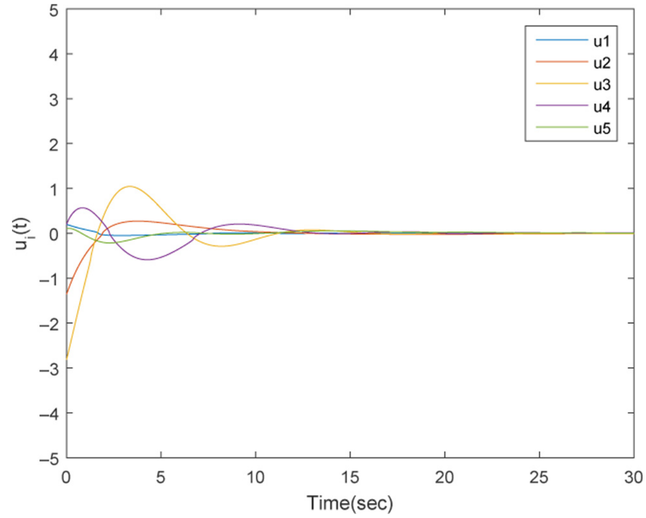


Figure 13 The sliding surface s_i of each agent

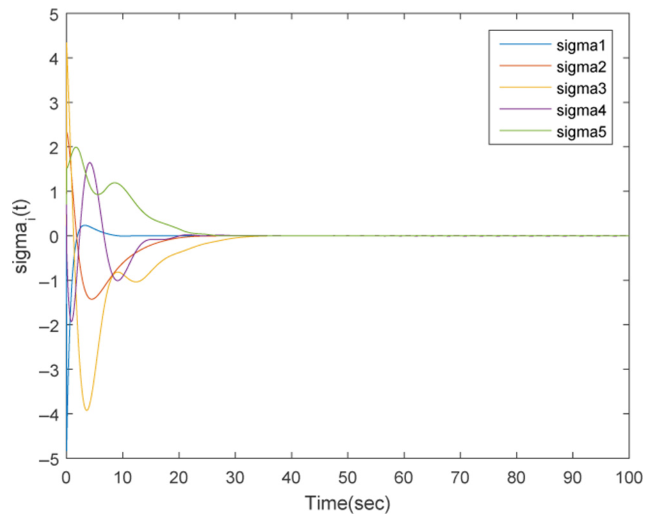


Figure 14 The position x_i of each agent

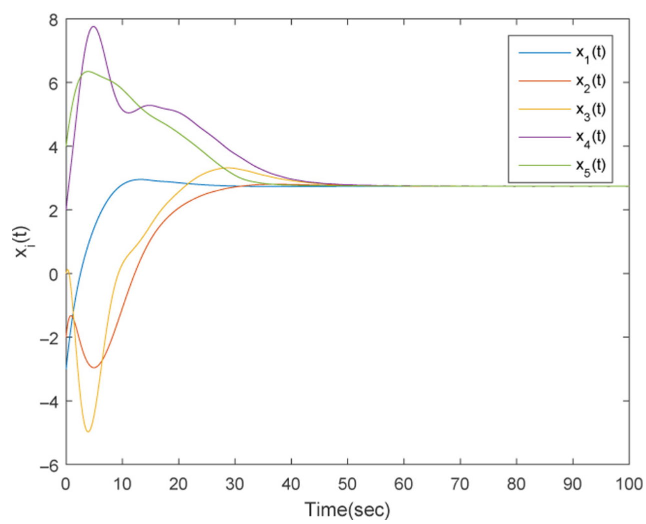


Figure 15 The velocity v_i of each agent

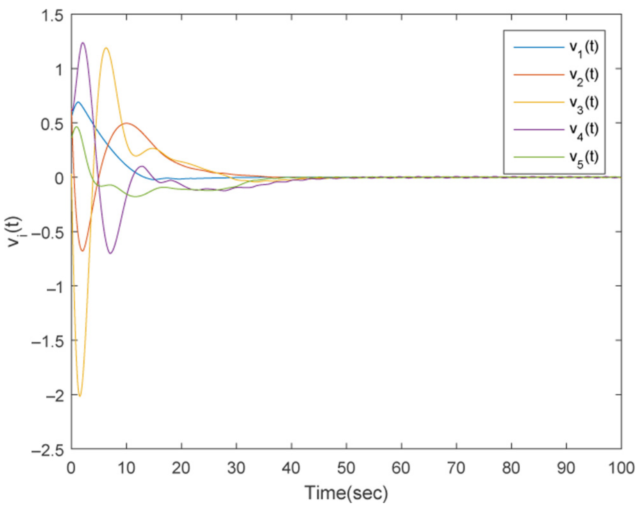


Figure 16 The input u_i of each agent

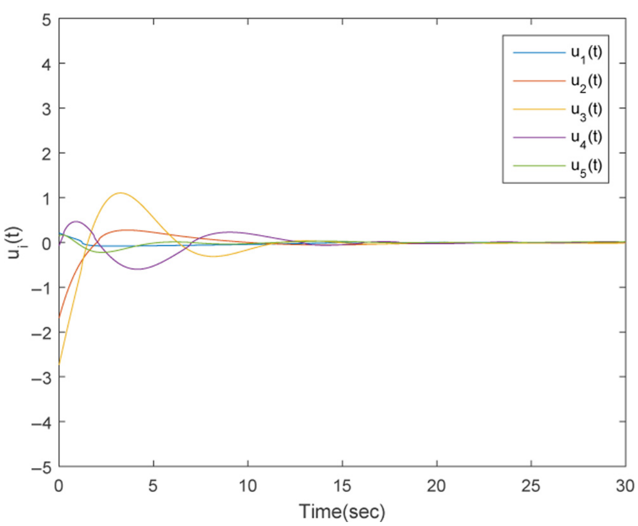
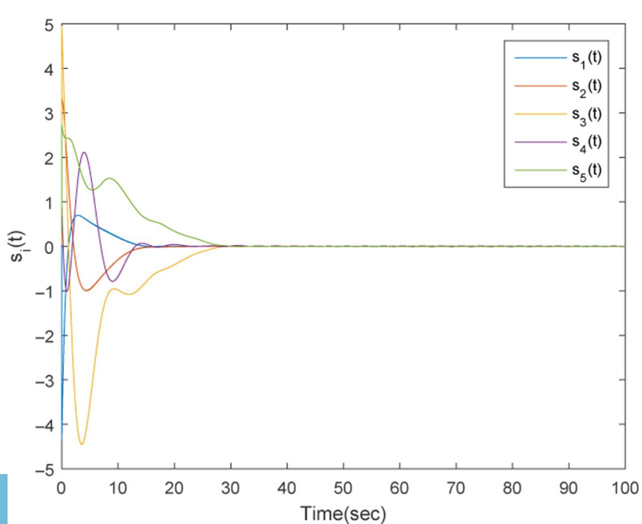


Figure 17 The sliding surface s_i of each agent



verify the effectiveness of the theoretical analysis. In the future, we will consider high-order MAS with mismatched uncertainties in directed networks, which are more challenging.

References

Chang, Y.H., Chang, C.W., Chen, C.L. and Tao, C.W. (2012), "Fuzzy sliding-mode formation control for multirobot systems: design and implementation", *IEEE Transactions on Systems, Man, and Cybernetics. Part B, Cybernetics : A Publication of the Ieee Systems, Man, and Cybernetics Society*, Vol. 42 No. 2, pp. 444-457.

Cheng, L., Wang, H.L., Hou, Z.G. and Tan, M. (2016), "Reaching a consensus in networks of high-order integral agents under switching directed topologies", *International Journal of Systems Science*, Vol. 47 No. 8, pp. 1966-1981.

Cheng, L., Hou, Z.G., Tan, M., Lin, Y.Z. and Zhang, W.J. (2010), "Neural-Network-Based Adaptive Leader-Following control for multiagent systems with uncertainties", *IEEE Trans. Neural Netw*, Vol. 21 No. 8, pp. 1351-1358.

Dong, X.W. and Hu, G.Q. (2016), "Time-varying formation control for general linear multi-agent systems with switching directed topologies", *Automatica*, Vol. 73, pp. 47-55.

Han, T., Guan, Z.H., Liao, R.Q., Chen, J., Chi, M. and He, D.X. (2017), "Distributed finite-time formation tracking control of multi-agent systems via FTSMC approach", *IET Control Theory & Applications*, Vol. 11 No. 15, pp. 2585-2590.

Hong, Y.G., Hu, J.P. and Gao, L.X. (2006), "Tracking control for multi-agent consensus with an active leader and variable topology", *Automatica*, Vol. 42 No. 7, pp. 1177-1182.

Li, Z.K., Liu, X.D., Ren, W. and Xie, L.H. (2013), "Distributed tracking control for linear multiagent systems with a leader of bounded unknown input", *IEEE Transactions on Automatic Control*, Vol. 58 No. 2, pp. 518-523.

Lin, P. and Jia, Y.M. (2009), "Consensus of second-order discrete-time multi-agent systems with nonuniform time-delays and dynamically changing topologies", *Automatica*, Vol. 45 No. 9, pp. 2154-2158.

Liu, C.L. and Tian, Y.P. (2009), "Formation control of multi-agent systems with heterogeneous communication delays", *International Journal of Systems Science*, Vol. 40 No. 6, pp. 627-636.

Liu, J., Yu, Y., Wang, Q. and Sun, C.Y. (2017), "Fixed-time event-triggered consensus control for multi-agent systems with nonlinear uncertainties", *Neurocomputing*, Vol. 260, pp. 497-504.

Li, J.P., Yang, Y.N., Hua, C.C. and Guan, X.P. (2016), "Fixed-time backstepping control design for high-order strict-feedback non-linear systems via terminal sliding mode", *IET Control Theory & Applications*, Vol. 11 No. 8, pp. 1184-1193.

Li, T. and Zhang, J.F. (2010), "Consensus conditions of multi-agent systems with time-varying topologies and stochastic communication noises", *IEEE Transactions on Automatic Control*, Vol. 55 No. 9, pp. 2043-2057.

Lu, K., Xia, Y., Zhu, Z. and Basin, M.V. (2012), "Sliding mode attitude tracking of rigid spacecraft with disturbances", *Journal of The Franklin Institute*, Vol. 349 No. 2, pp. 413-440.

Mu, C.X., Xu, W. and Sun, C.Y. (2016), "On switching manifold design for terminal sliding mode control", *Journal of The Franklin Institute*, Vol. 353 No. 7, pp. 1553-1572.

Olfati-Saber, R. (2006), "Flocking for multi-agent dynamic systems: algorithms and theory", *IEEE Transactions on Automatic Control*, Vol. 51 No. 3, pp. 401-420.

Olfati-Saber, R. and Murray, R.M. (2004), "Consensus problems in networks of agents with switching topology and time-delays", *IEEE Transactions on Automatic Control*, Vol. 49 No. 9, pp. 1520-1533.

Ren, W. and Atkins, E. (2007), "Distributed multi-vehicle coordinated control via local information exchange", *International Journal of Robust and Nonlinear Control*, Vol. 17 Nos 10/11, pp. 1002-1033.

Ren, C.E. and Chen, C.P. (2015), "Sliding mode leader-following consensus controllers for second-order non-linear multi-agent systems", *IET Control Theory & Applications*, Vol. 9 No. 10, pp. 1544-1552.

Sam, Y.M., Osman, J.H. and Ghani, M.R.A. (2004), "A class of proportional-integral sliding mode control with application to active suspension system", *Syst. Control Lett*, Vol. 51 Nos 3/4, pp. 217-223.

Sun, C.Y., Wang, Q. and Yu, Y. (2017), "Robust output containment control of multi-agent systems with unknown heterogeneous nonlinear uncertainties in directed networks", *International Journal of Systems Science*, Vol. 48 No. 6, pp. 1173-1181.

Wang, Y.P., Cheng, L., Hou, Z.G., Tan, M. and Wang, M. (2014), "Containment control of multi-agent systems in a noisy communication environment", *Automatica*, Vol. 50 No. 7, pp. 1922-1928.

Wang, Q., Yu, Y. and Sun, C.Y. (2018), "Distributed event-based consensus control of multi-agent system with matching nonlinear uncertainties", *Neurocomputing*, Vol. 272, pp. 694-702.

Xu, W.Y., Cao, J.D., Yu, W.W. and Lu, J.Q. (2014), "Leader-following consensus of non-linear multi-agent systems with jointly connected topology", *IET Control Theory & Applications*, Vol. 8 No. 6, pp. 432-440.

Yan, Y., Galias, Z., Yu, X.H. and Sun, C.Y. (2016), "Euler's discretization effect on a twisting algorithm based sliding mode control", *Automatica*, Vol. 68, pp. 203-208.

Yang, J., Li, S.H. and Yu, X.H. (2013), "Sliding-mode control for systems with mismatched uncertainties via a disturbance observer", *IEEE Transactions on Industrial Electronics*, Vol. 60 No. 1, pp. 160-169.

Yang, J., Su, J.Y., Li, S.H. and Yu, X.H. (2014), "High-Order mismatched disturbance compensation for motion control systems via a continuous dynamic Sliding-Mode approach", *IEEE Transactions on Industrial Informatics*, Vol. 10 No. 1, pp. 604-614.

Yu, W.W., Chen, G.R. and Cao, M. (2010), "Distributed leader? Follower flocking control for multi-agent dynamical systems with time-varying velocities", *Syst. Control Lett*, Vol. 59 No. 9, pp. 543-552.

Yu, S.H. and Long, X.J. (2015), "Finite-time consensus for second-order multi-agent systems with disturbances by integral sliding mode", *Automatica*, Vol. 54, pp. 158-165.

Zhang, J.H., Liu, X.W., Xia, Y.Q., Zuo, Z.Q. and Wang, Y.J. (2016), "Disturbance observer-based integral sliding-mode control for systems with mismatched disturbances", *IEEE Transactions on Industrial Electronics*, Vol. 63 No. 11, pp. 7040-7048.

Zou, A.M., Kumar, K.D. and Hou, Z.G. (2013), "Distributed consensus control for multi-agent systems using terminal sliding mode and Chebyshev neural networks", *International Journal of Robust and Nonlinear Control*, Vol. 23 No. 3, pp. 334-357.

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